MATH 0332 Contemporary Math

July 6, 2020

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Homework 10.3

Using Permutations and Combinations

1.

Evaluate the expression.

$_6P_5$

The solution is 720 .

$$nPr = \frac{n!}{(n-r)!} = \frac{6!}{(6-5)!} = 720$$

Evaluate the expression.

$_8C_5$

$_8C_5$ = 56

2.

$$nCr = \frac{n!}{(n-r)!r!} = \frac{8!}{(8-5)!5!} = 56$$

3.

Determine the number of permutations (arrangements) possible of 9 things taken 2 at a time.

The answer is 72 .

n – is the largest number $9P2$ = 72

r – is the smallest number

4.

Find the number of combinations (subsets) of 8 things taken 3 at a time.

The answer is 56 combinations.

n – is the largest number $9P2$ = 56

r – is the smallest number

5.

Determine whether the object is a permutation or a combination.

a 10-digit telephone number (including area code)

Choose the correct answer below.

- A. This is a permutation because repetition is not allowed and the order of the items matters.
- B. This is a combination because repetition is not allowed and the order of the items does not matter.
- C. This is neither a permutation nor a combination because repetition is allowed.
- D. There is not enough information given to make a decision.

Permutations are an ordered arrangement of items that occur when

- No item is used more than once
- The order of arrangement makes a difference

Combinations are an ordered arrangement of items that occur when

- No item is used more than once
- The order of arrangement makes NO difference

6.

A contractor builds homes of 11 different models and presently has 6 lots to build on. In how many different ways can he arrange homes on these lots? Assume 6 different models will be built.

The answer is 332640 arrangements.

Since all 6 lots will have different models being built, order DOES matter

$$11P6 = 332640$$

7.

How many ways can a president, vice-president, and secretary be chosen from a committee of 10 people?

The number of ways to choose a president, vice-president, and secretary is 720 .

There are 3 positions available, only one person can fill each position, order DOES matter

$$10P3 = 720$$

8.

How many counting numbers have four distinct nonzero digits such that the sum of the four digits is 10?

There are 24 such counting numbers.

Nonzero digits = 1, 2, 3, 4, 5, 6, 7, 8, 9

1 + 2 + 3 + 4 = 10

$$\frac{4!}{(4-4)!} = \frac{4!}{0!} = 24$$

9.

An electronics store receives a shipment of 40 graphing calculators, including 5 that are defective. Four of the calculators are selected to be sent to a local high school. How many of these selections will contain no defective calculators?

The number of ways to choose all selections that contain no defective calculators is
52360 .

Nothing signifies that order matters – Combination

1st take the defective calculators out of the equation 40 – 5 = 35

2nd input numbers into a combination formula $35C4 = \frac{35!}{4!(35-4)!}$

10.

A standard 52-card deck contains four jacks, twelve face cards, thirteen hearts (all red), thirteen diamonds (all red), thirteen spades (all black), and thirteen clubs (all black). Of the 133,784,560 different seven-card hands possible, decide how many would consist of the following.
(a) all hearts
(b) all red cards
(c) all jacks

(a) There are 1716 ways to have a hand with all hearts.
(Simplify your answer.)

(b) There are 657800 ways to have a hand with all red cards.
(Simplify your answer.)

(c) There are 0 ways to have a hand with all jacks.
(Simplify your answer.)

Order does not matter – Combination

(a) There are 13 cards total, each hand has 7 cards
13C7 = 1716

(b) There are 26 cards total, each hand has 7 cards
26C7 = 657800

(c) There are 4 cards total, each hand has 7 cards
This is impossible. There is no way to have a 7 card hand with 4 cards.

11.

To win at LOTTO in one state, one must correctly select 6 numbers from a collection of 56 numbers (1 through 56). The order in which the selection is made does not matter. How many different selections are possible?

There are 32468436 different LOTTO selections.

Order does not matter – Combination

$_{56}C_6 = 32468436$

12.

Subject identification numbers in a certain scientific research project consist of two digits followed by three letters and then two more digits. Assume repetitions are not allowed within any of the three groups, but digits in the first group of two may occur also in the last group of two. How many distinct identification numbers are possible?

There are 126360000 distinct identification numbers possible.

Order does matter - Permutation

2 digits out of 0-9 $_{10}P_2 = 90$

3 letters out of A-Z $_{26}P_3 = 15{,}600$

2 digits out of 0-9 $_{10}P_2 = 90$

Now use the fundamental counting principle

90 x 15,600 x 90 = 126,360,000

13.

Television and radio stations use four call letters starting with W or K, such as WXYZ or KRLD. Assuming no repetitions in the second to fourth letters, how many four-letter sets are possible using either W or K and only the letters G to V? (Count all possibilities even though, practically, some may be inappropriate.)

There are 6720 four-letter sets that can be formed.

W or K is the first letter, leaving only 2 options

G-V is available for the next 3 letters $_{16}P_3 = 3360$

Now use the fundamental counting principle 2 x 3360 = 6720

14.

A baseball team has 8 pitchers, who only pitch, and 9 other players, all of whom can play any position other than pitcher. For Saturday's game, the coach has not yet determined which 9 players to use nor what the batting order will be, except that the pitcher will bat last. How many different batting orders may occur?

There are 2903040 different batting orders.

There are 8 possible pitchers, who have the possibility of batting last $8C1 = 8$

There are 9 other players, and 8 batting positions open $9P8 = 362{,}880$

Now use the fundamental counting principle $8 \times 362{,}880 = 2{,}903{,}040$

15.

In how many ways could 18 people be divided into five groups containing, respectively, 1, 5, 4, 6, and 2 people?

The groups can be chosen in 1,543,782,240 ways.

$18C1 = 18$	$8C6 = 28$	Now use the fundamental counting principle
$17C5 = 6{,}188$	$2C2 = 1$	$18 \times 6{,}188 \times 495 \times 28 \times 1$
$12C4 = 495$		$= 1{,}543{,}782{,}240$

16.

In how many ways could five people be divided into two groups of two people and one group of one person?

Five people could be divided into two groups of two people and one group of one person 15 ways.

(Type a whole number.)

$5C2 = 10$ $3C2 = 3$ $1C1 = 1$

Now use the fundamental counting principle $10 \times 3 \times 1 = 30$

Since the order is not important, identical answers must be taking into account. Divide 30 by 2

$$\frac{30}{2!} = 15$$

17.

How many possible 5-card hands from a standard 52-card deck would consist of the following cards?

(a) two clubs and three non-clubs
(b) one face card and four non-face cards
(c) three black cards, one heart, and one diamond

(a) There are 712,842 five-card hands consisting of two clubs and three non-clubs.
(Type a whole number.)

(b) There are 1,096,680 five-card hands consisting of one face card and four non-face cards.
(Type a whole number.)

(c) There are 439,400 five-card hands consisting of three black cards, one heart, and one diamond.
(Type a whole number.)

(a)

There are 13 clubs, you need 2 $\quad$ 52-13=39 $\quad$ There are 39 non-clubs, you need 3

$_{13}C_2$ $\quad$ $_{39}C_3$

Use the fundamental counting principle. $\quad$ $_{13}C_2 \times {}_{39}C_3 = 712{,}842$

(b)

There are 12 face cards, you need 1 $\quad$ 52-12=40 $\quad$ There are 40 non-face cards. You need 4

$_{12}C_1$ $\quad$ $_{40}C_4$

Use the fundamental counting principle. $\quad$ $_{12}C_1 \times {}_{40}C_4 = 1{,}096{,}680$

(c)

There are 26 black cards, you need 3 $\quad$ $_{26}C_3$

There are 13 heart cards, you need 1 $\quad$ $_{13}C_1$

There are 13 diamond cards, you need 1 $\quad$ $_{13}C_1$

Use the fundamental counting principle. $\quad$ $_{26}C_3 \times {}_{13}C_1 \times {}_{13}C_1 = 439{,}400$

18.

How many cards must be drawn (without replacement) from a standard deck of 52 to guarantee that twelve of the cards will be of the same suit.

To guarantee that twelve of the cards will be of the same suit, 45 cards must be drawn.

There are 4 suits in a standard deck of 52 cards

Suppose that every 4 cards drawn represents a card from each suit, then it is possible that you have the exact same number of cards from each suit

If 44 cards have been drawn, and there is not a set of 12 from one suit, then the hand consists of only 11 cards from each suit. 44 / 4 = 11

If this is the case, drawing an additional card guarantees that there will be twelve cards from one suit. 44 + 1 = 45

19.

The trifecta at most racetracks consists of selecting the first-, second-, and third-place finishers in a particular race in their proper order. If there are five entries in the trifecta race, how many tickets must you purchase to guarantee a win?

You must purchase 60 tickets.

Order does matter – Permutation

$_5P_3 = 60$

20.

The daily double at most race tracks consists of selecting the winning horse in both the first and second races. If the first race has 8 entries and the second race has 9 entries, how many daily double tickets must be purchased to guarantee a win?

How many tickets must be purchased?

72

Use the fundamental counting principle.

8 x 9 = 72