MATH 0332 Contemporary Math

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11.3

Conditional Probability and Events Involving "And"

1.

From the sample space S = {1, 2, 3, 4,..., 15} a single number is to be selected at random. Given event A, that the selected number is even, and event B, that the selected number is a multiple of 4, find P(A|B).

P(A|B) = 1 (Simplify your answer. Type an integer or a fraction.)

S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15} n(S) = 15

A: The selected number is even = {2, 4, 6, 8, 10, 12, 14} n(A) = 7

B: The selected number is a multiple of 4 = {4, 8, 12} n(B) = 3

$A \cap B$ = {4, 8, 12} n($A \cap B$) = 3

Use the formula: P(A I B) = $\frac{P(A \text{ and } B)}{P(B)}$

$\frac{3}{3}$ = 1

2.

From the sample space S = {1,2,3,4,...,15}, a single number is to be selected at random. Given the following events, find the indicated probability.

A: The selected number is even.
B: The selected number is a multiple of 4.
C: The selected number is a prime number.

P(B|A)

P(B|A) = $\frac{3}{7}$

(Simplify your answer.)

C is insignificant to this problem

S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15} n(S) = 15

A: The selected number is even = {2, 4, 6, 8, 10, 12, 14} n(A) = 7

B: The selected number is a multiple of 4 = {4, 8, 12} n(B) = 3

$A \cap B$ = {4, 8, 12} n($A \cap B$) = 3

Use the formula: P(B | A) = $\frac{n(A \text{ and } B)}{n(A)}$

P(B | A) = $\frac{3}{7}$

3.

From the sample space S = {1, 2, 3, 4,..., 15} a single number is to be selected at random. Given event A, that the selected number is even, and event C, that the selected number is a prime number, find P(C|A).

P(C|A) = $\frac{1}{7}$ (Simplify your answer. Type an integer or a fraction.)

S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15} n(S) = 15

A: The selected number is even = {2, 4, 6, 8, 10, 12, 14} n(A) = 7

C: The selected number is a prime number = {2, 3, 5, 7, 11, 13} n(C) = 6

$A \cap C$ = {2} n($A \cap C$) = 1

Use the formula: P(C | A) = $\frac{n(A \text{ and } C)}{n(A)}$

P(C | A) = $\frac{1}{7}$

4.

Given a family with three children, find the probability of the event.

All are girls.

The probability that all of the children are girls is $\frac{1}{8}$.

(Simplify your answer. Type an integer or a fraction.)

S = {ggg, ggb, gbg, gbb, bgg, bgb, bbg, bbb} n(S) = 8

A: The event that all children are girls = {ggg} n(A) = 1

Use the formula: $\frac{n(A)}{n(S)} = \frac{1}{8}$

5.

Given a family with four children, find the probability of the event.

The youngest is a girl, given that there are at least two girls.

The probability that the youngest is a girl, given that there are at least two girls is $\frac{7}{11}$.

(Simplify your answer. Type an integer or a fraction.)

Use the formula: P(A I B) = $\frac{P(A\ and\ B)}{P(B)}$

S = {gggg, gggb, ggbg, ggbb, gbgg, gbgb, gbbg, gbbb, bggg, bggb, bgbg, bgbb, bbgg, bbgb, bbbg, bbbb}

n(S) = 16

A: The youngest is a girl = {gggg, ggbg, gbgg, bgggg, gbbg, bgbg, bbgg, bbbg}

n(A) = 8

B: That there are at least two girls = {gggg, gggb, ggbg, gbgg, bggg, ggbb, gbgb, bggb, gbbg, bgbg, bbgg}

n(A) = 11

$A \cap B$ = {gggg, ggbg, gbgg, bggg, gbbg, bgbg, bbgg}

n($A \cap B$) = 7

P(A I B) = $\frac{7}{11}$

6.

For the following experiment, determine whether the two given events are independent.

A fair coin is tossed twice. The events are "head on the first" and "head on the second."

Choose the correct answer below.

- A. The two events are independent because the outcome of the second coin toss does not depend on the outcome of the first coin toss.
- B. The two events are not independent because the coin came up heads both times.
- C. The two events are independent because each coin toss cannot have any outcomes in common.
- D. The two events are not independent because the outcome of the second coin toss depends on the outcome of the first coin toss.

Two events are independent events if knowledge about the occurrence of one of them has no effect on the probability of the other one occurring.

7.

Determine whether the two given events are independent.

A pair of dice are rolled. The events are "odd on the first" and "even on the second."

The two events are independent.

The events are independent because the first event occurring has no effect on the second event occurring.

8.

A pet store has 15 puppies, including 5 poodles, 5 terriers, and 5 retrievers. If Rebecka and Aaron, in that order, each select one puppy at random with replacement (they may both select the same one), find the probability that they both select a poodle.

The probability is $\frac{1}{9}$.

(Type an integer or a simplified fraction.)

First define the two events. Lets say that

A = Rebecka chooses a poodle

B = Aaron chooses a poodle

These are independent events

Use the formula P(A and B) = P(A) x P(B)

$P(A) = \frac{5}{15} = \frac{1}{3}$

$P(B) = \frac{5}{15} = \frac{1}{3}$ (since selection is done with replacement)

$\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

9.

A pet store has 12 puppies, including 3 poodles, 4 terriers, and 5 retrievers. If Rebecka and Aaron, in that order, each select one puppy at random without replacement, find the probability that Aaron selects a retriever, given that Rebecka selects a poodle.

The probability is $\frac{5}{11}$.

(Type an integer or a fraction.)

A = Aaron selects a retriever

B= Rebecka selects a poodle

Once Rebecka has made her choice, there are 11 puppies available for Aaron

Out of those puppies, there are 5 retrievers

$P(A \mid B) = \frac{5}{11}$

10.

A pet store has 6 puppies, including 2 poodles, 3 terriers, and 1 retriever. Rebecka and Aaron, in that order, each select one puppy at random without replacement. Find the probability that Aaron selects a retriever, given Rebecka selects a retriever.

The probability is 0.

(Type an integer or a simplified fraction.)

A = Aaron selects a retriever

B = Rebecka selects a retriever

Once Rebecka has selected her choice, there are 0 retrievers left for Aaron to pick

11.

Let two cards be dealt successively, without replacement, from a standard 52-card deck. Find the probability of the event.

club dealt second, given a club dealt first

The probability that the second is a club, given that the first is a club is $\frac{4}{17}$.

(Simplify your answer. Type an integer or a fraction.)

There are 52 cards in a deck, 4 suits in total, and each suit has 13 cards.

Use the formula: P(E) = $\frac{number\ of\ favorable\ outcomes}{total\ number\ of\ outcomes} = \frac{n(E)}{n(S)}$

After the first card is drawn, there are 51 cards left in the deck

n(S) = 51

After the first card is drawn, a club, there are 12 clubs left in the deck

n(E) = 12

P(E) = $\frac{12}{51} = \frac{4}{17}$

12.

Let two cards be dealt successively, without replacement, from a standard 52-card deck. Find the probability of the event.

two kings

The probability of drawing two kings is $\frac{1}{221}$.

(Simplify your answer. Type an integer or a fraction.)

Use the formula: P(A and B) = P(A) x P(B l A)

Use the formula: P(E) = $\frac{number\ of\ favorable\ outcomes}{total\ number\ of\ outcomes} = \frac{n(E)}{n(S)}$

A is the event that the first card drawn is a king

n(S) = 52 (total of cards in deck)

n(A) = 4 (total of kings in deck)

P(A) = $\frac{n(A)}{n(S)} = \frac{4}{52} = \frac{1}{13}$

After the first card is drawn, there are 51 cards left $\qquad$ n(S) = 51

After the first king is drawn, there are 3 kings left $\qquad$ n(B) = 3

P(B l A) = $\frac{n(B\ l\ A)}{n(S)} = \frac{3}{51} = 1/17$

13.

Let two cards be dealt successively, without replacement, from a standard 52-card deck. Find the probability of the event.

The first card is a seven and the second is a seven.

The probability that the first card is a seven and the second is a seven is $\frac{1}{221}$.

(Simplify your answer. Type an integer or a fraction.)

Use the formula: P(E) = $\frac{number\ of\ favorable\ outcomes}{total\ number\ of\ outcomes} = \frac{n(E)}{n(S)}$

n(S) = 52

n(A) = 4

P(A) = $\frac{4}{52} = \frac{1}{13}$

After the first card is drawn, there will be 51 cards left in the deck

n(S) = 51

After the first seven is drawn, there will be 3 left in the deck

n(B l A) = 3

P(B l A) = $\frac{3}{51} = \frac{1}{17}$

Use the formula: P(A and B) = P(A) x P(B l A)

$\frac{1}{13}\ x\ \frac{1}{17} = \frac{1}{221}$

14.

Use the formula $P(B|A) = \frac{n(A \text{ and } B)}{n(A)}$ to find the probability P(king|face card) when a single card is drawn from a standard 52-card deck.

P(king|face card) = $\frac{1}{3}$ (Simplify your answer. Type an integer or a fraction.)

P(kinglface card) = $\frac{n(king\ and\ face\ card)}{n(face\ card)}$

n(face card) = 12

n(king and face card) = 4

P(kinglface card) = $\frac{4}{12} = \frac{1}{3}$

15.

Given events A and B within the sample space S, the following formula can be used to compute conditional probabilities.

$$P(B|A) = \frac{n(A \text{ and } B)}{n(A)}$$

Use this result to find the probability P(black|spade) when a single card is drawn from a standard 52-card deck.

P(black|spade) = 1 (Simplify your answer.)

P(blacklspade card) = $\frac{n(spade\ and\ black)}{n(spade)}$

n(spade) = 13

n(spade and black) = 13

P(kinglface card) = $\frac{13}{13}$ = 1

16.

Three families each have three sons and no daughters. Assuming boy and girl babies are equally likely, what is the probability of this event?

The probability is $\frac{1}{512}$.

(Type an integer or a simplified fraction.)

The probability of having a son versus a daughter is $\frac{1}{2}$

The probability of have three sons and no daughters in a family is $\frac{1}{2} \text{ x } \frac{1}{2} \text{ x } \frac{1}{2}$

The probability of three families having three sons and no daughters is $\frac{1}{2} \text{ x } \frac{1}{2} \text{ x } \frac{1}{2} \text{ x } \frac{1}{2} \text{ x } \frac{1}{2} \text{ x } \frac{1}{2} \text{ x } \frac{1}{2} \text{ x } \frac{1}{2} \text{ x } \frac{1}{2}$

$= \frac{1}{512}$

17.

Three dice are tossed. What is the probability that the numbers shown will all be different?

The probability that the numbers shown will all be different is $\frac{5}{9}$.

(Type an integer or a simplified fraction.)

Use the formula: P(E) = $\frac{number\ of\ favorable\ outcomes}{total\ number\ of\ outcomes} = \frac{n(E)}{n(S)}$

6 x 6 x 6 = 216

n(S) = 216

There are 6 possibilities for the first die

Since 1 possibility has been taken, there are 5 possibilities for the second die

Since 2 possibilities have been taken, there are 4 possibilities for the third die

6 x 5 x 4 = 120

n(E) = 120

$\frac{n(E)}{n(S)} = \frac{120}{216} = \frac{5}{9}$